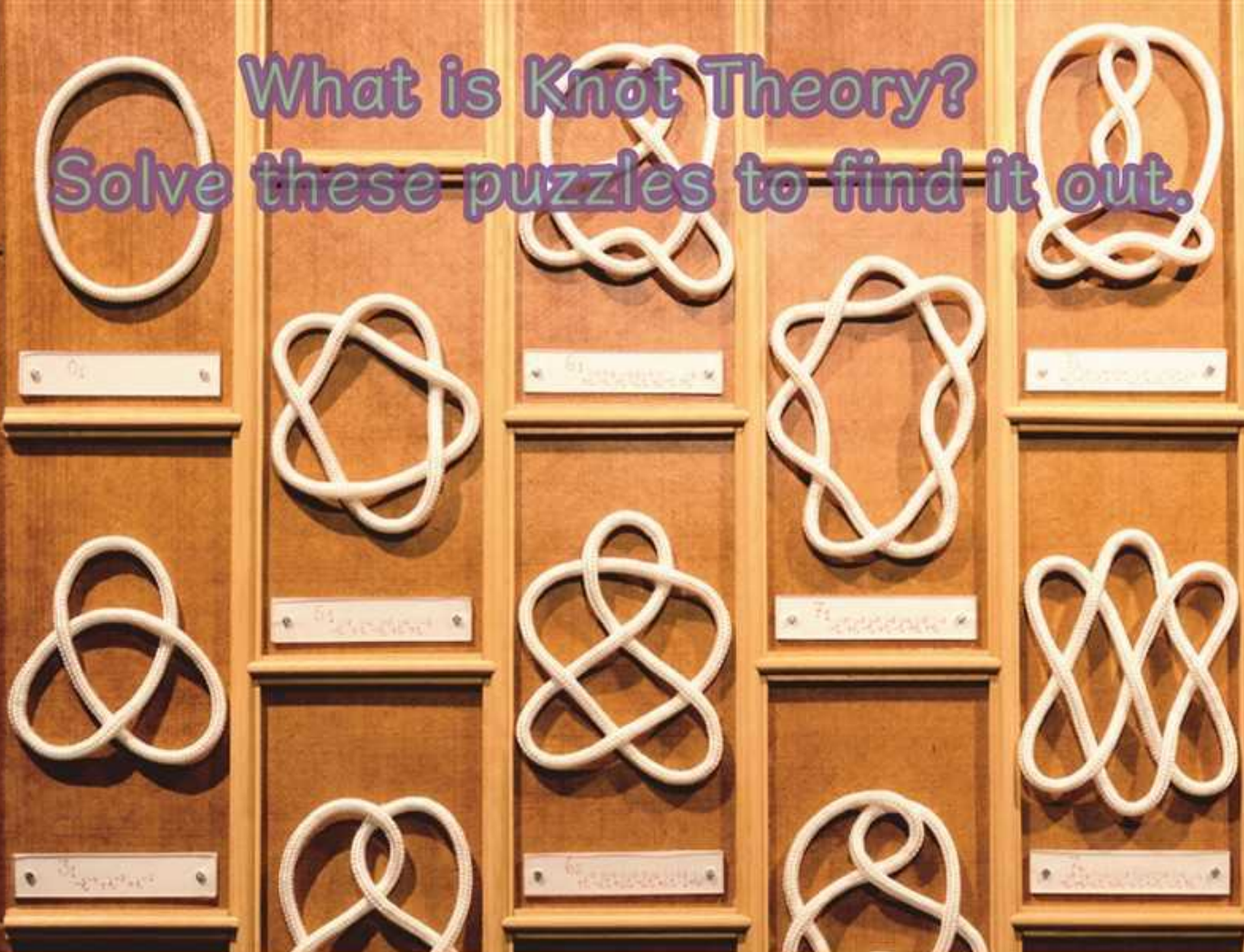


What is Knot Theory?

Solve these puzzles to find it out.

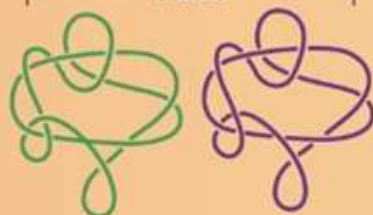


Which knot in each of the following pairs can be unravelled into a circle without opening the loop?

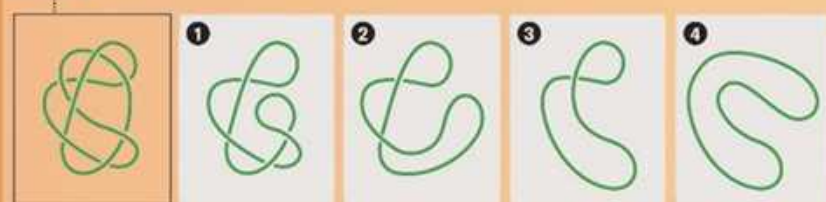
Pair 1



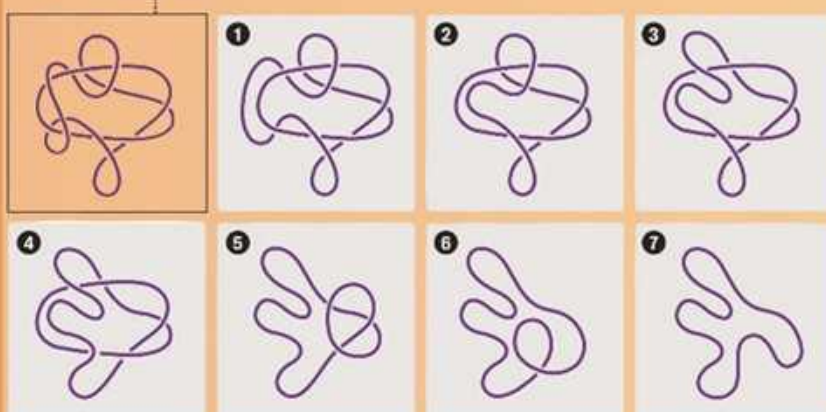
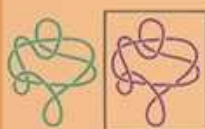
Pair 2



In pair 1, the knot on the left can be completely unravelled.



In pair 2, the knot on the right also does so.



Have you ever wrestled with a tangled pair of earphones or tied a stubborn shoelace? Knots are a part of our daily lives, found in everything from fishing nets to decorative ties.

But to a mathematician, a "knot" is something very specific: a single loop of string with its ends joined together, like an extension cord being plugged into itself. The fascinating study of these tangled loops is called *Knot Theory*.

What is Knot Theory?

The main question in Knot Theory is: can a complex-looking knot be untangled into a simple circle (called an "unknot") without cutting the string?

Some knots are deceptively simple. For example, the trefoil knot, which looks like a three-leaf clover, is the simplest "true" knot. It is impossible to untie it without breaking the rope. Others, despite looking complicated, can be wiggled and pulled until they magically untangle into a perfect circle.

To tell these knots apart, mathematicians use clever tools. They study 2D drawings of the knot and apply special rules, called Reidemeister moves, to see how it can be simplified. They also give knots special names, like the "figure-eight," and use advanced techniques to prove if a knot is truly "knotted."



The trefoil knot is the simplest mathematical knot. It is impossible to untie it without breaking the 'rope'.

Examples of traditional Celtic knots



A Knotty History

Humans have been tying knots since prehistoric times, first for practical purposes like sailing, fishing, and building. Over time, knots have also evolved into beautiful artistic forms and deep cultural symbols. The closed loops studied by mathematicians, in particular, show up as a recurring motif in ancient Chinese and Celtic artwork that dates back centuries.

Why Knots Matter Today

Knot theory isn't just a mathematical puzzle. It has powerful real-world applications. Scientists use it to understand how:

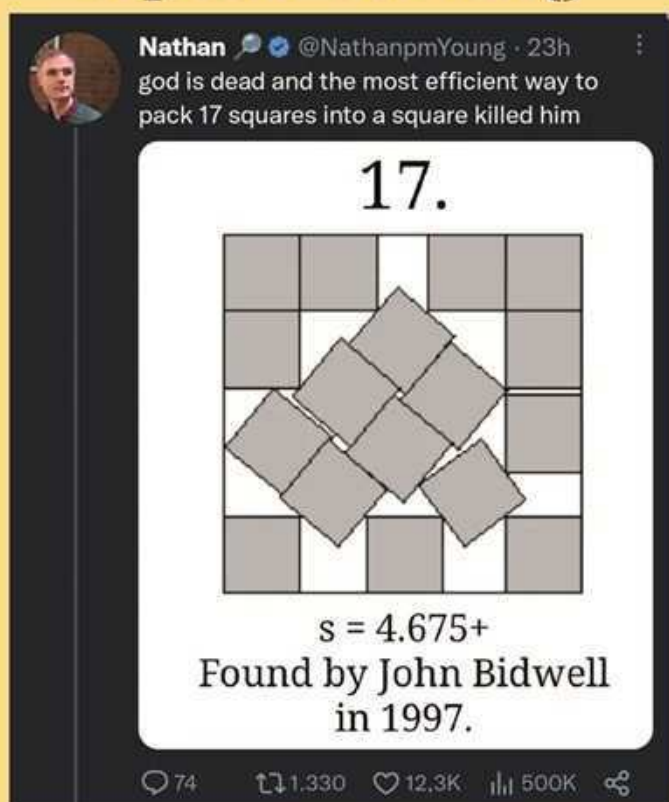
- DNA twists and tangles inside our cells,
- proteins fold into complex shapes to function,
- new materials can be designed at a molecular level.

So the next time you untangle your earphones, just remember: you're secretly dabbling in mathematical research!

**WHY DO
CALCULATORS
MAKE GREAT
FRIENDS?**

YOU CAN ALWAYS COUNT ON THEM!

Square Packing



If you have some unit squares, and want to put them in a larger square, how big does the larger square have to be?

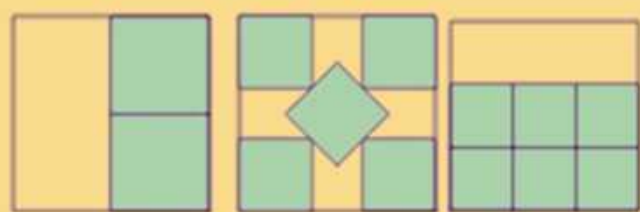
Some answers are straightforward:

If you have 1 unit square it fits in a 1×1 square.

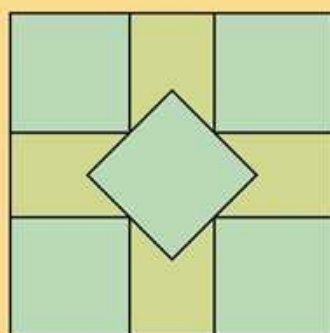
If you have 4 unit squares, they fit in a 2×2 square.



But what if you only have 2 or 3 unit squares? Can they fit into something smaller than a 2×2 square?



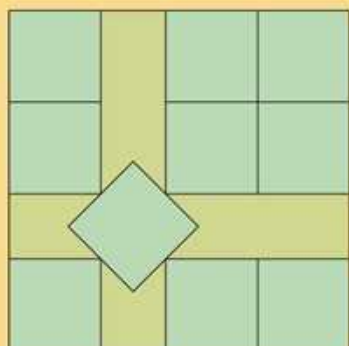
It turns out they can't. Even just 2 unit squares take up too much space if you rotate them around, so you're stuck with a 2×2 square. But that rotation does help if you have 5 unit squares.



The 5 unit squares can fit into a square that is about 2.7 units on each side — technically it's $2 + (\sqrt{2})/2$.

But 6 squares isn't so efficient — it fits into a 3×3 square with room left over, and nothing smaller will do.

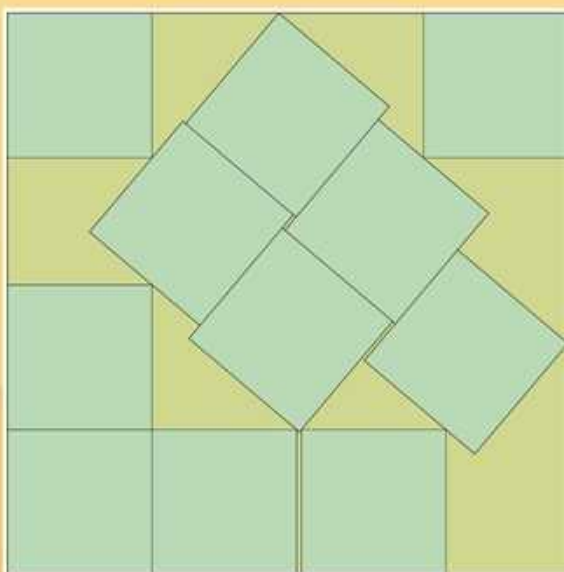
The same goes for 7 or 8 squares, and then 9 squares fit perfectly into that 3×3 with no room for anything else.



Based on this pattern, though, you might guess that if you have 10 squares, then you can do some rotation to get something smaller than jumping all the way to 4×4 , and you'd be right. As a Creative Commons picture adapted by Maksim shows, these 10 blocks fit into a square that's only about 3.7 on each side — technically it's $3 + (\sqrt{2})/2$.

We'll try one more. What about 11 squares? Can those fit into something smaller than a 4×4 square?

The answer is yes, but exactly how much smaller is still unknown. Walter Trump, a German high school teacher, found an efficient packing for 11 squares back in 1979, where they fit into a square that was only 3.87 on each side as seen in his creative commons picture, but there's a chance that it could be smaller, closer to 3.79.



John Bidwell found a solution for packing 17 squares into a square of side 4.67 while he was an undergraduate by rotating different amounts. See the figure at the top to see the packing.

Source: <https://threesixty360.wordpress.com/>

Game Time

1. Board and Goal

On a 6×6 grid, the starting cell in the top-left corner contains the number "1", and the bottom-right cell is marked with a "★". The goal is to move from "1" to "★".

2. Movement Rules

On each move, you may only travel up, down, left, or right, and the number of squares you jump must exactly match the number in your current cell.

For example: If you are on a cell with the number 3, you must move 3 squares in one direction, landing on the fourth square (skipping two in between).

3. Restrictions

You cannot jump outside the grid.

Game 1:

1	2	5	1	4	2
3	2	3	2	1	3
3	4	4	1	2	2
2	2	1	4	1	2
3	1	2	3	3	3
1	2	2	3	2	★

Game 2:

2	4	1	3	2	1
3	1	4	2	3	4
1	2	3	1	2	3
4	3	1	2	4	1
2	1	4	3	1	2
3	4	2	1	3	★

Solution:

★	2	3	2	2	1
3	3	3	2	1	3
2	1	4	1	2	2
2	2	1	4	4	3
3	2	3	2	3	3
1	4	2	5	1	1

